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Engineering Notes

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Terminal Guidance Law Based on Proportional Navigation

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Introduction

A TOPIC of classical interest in atmospheric flight dynamics of a landing aircraft or lifting-body vehicle is the synthesis of flight path from an arbitrary initial point to a terminal point with a desired final velocity vector direction. This problem has been extensively analyzed in the published literature.[1–3] In a sense, similar problems can be considered for a mobile body such as an automobile, mobile robot, floating point, etc.

Proportional navigation is a well-known homing interception guidance law[4–6] for which the velocity vector (heading) of the interceptor is turned at a rate proportional with a navigation ratio λ to the rotation rate of the line joining the interceptor and moving target, which is the line of sight. As a rule, for the intercept trajectories the navigation ratio is held fixed. It is evident that the proportional navigation is acceptable for the nonmoving targets. In this case, the navigation ratio can be considered as a control variable, which depends on the current and desired final states. This Note presents a terminal guidance law to find a vehicle trajectory with the specified heading at the desired final point.

Classical Proportional Navigation

For the horizontal plane (Fig. 1) with the origin fixed at the final point, the kinematic equations of a vehicle are

$$\dot{r}_h = V_h \cos(\psi - \varphi) \tag{1a}$$

$$\dot{\varphi} = (V_h / r_h) \sin(\psi - \varphi) \tag{1b}$$

where V_h is the horizontal velocity, r_h is the horizontal range, φ is the line-of-sight angle, and ψ is the heading angle. The proportional guidance law for the turning rate is[4]

$$\dot{\psi} = \lambda_h \dot{\varphi} \tag{2}$$

where λ_h is a navigation ratio.

As known from the qualitative analysis of Eqs. (1–3) for arbitrary time-varying velocity,[6] in the case of $\lambda_h > 2$ the required turning acceleration of the vehicle is approaching zero near the final point.

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Plane Guidance

Without loss of generality, suppose that the OX axis is in the direction opposite to the desired final heading. We can assume that there is the exact motion corresponding to Eq. (2) and the navigation rate λ_h is a constant. After integrating, we have

$$\psi_f - \psi_0 = \lambda_h(\varphi_f - \varphi_0) \tag{3}$$

For the boundary values $\psi_f \equiv -\pi$, $\varphi_f \equiv 0$, we have

$$\pm\pi - \psi_0 = -\lambda_h \varphi_0 \tag{4}$$

Thus, the required navigation ratio λ_h is

$$\lambda_h = [\text{sign}(\varphi_0)\pi + \psi_0]/\varphi_0 \tag{5}$$

For $\lambda_h > 2$, the ideal (or unperturbed) trajectory must be passed through the final point with desired heading and $\dot{\psi} \to 0$. If $\lambda_h < 2$, then a forced turn of the vehicle to an acceptable angle ψ is needed. Because the ideal proportional navigation trajectories and lines of sight are not intersected, the initial and final velocity vectors must lie in different half-planes, which are formed by the lines of sight. In the opposite case, the forced turn is also needed.

The navigation ratio must be computed periodically by using the current state. For a compensation of unpredictable perturbations and control errors, we can use a margin for the navigation ratio, that is, an acceptable value should be no less than $\lambda_h > 2 + \Delta\lambda_h$, where $\Delta\lambda_h$ is the margin.

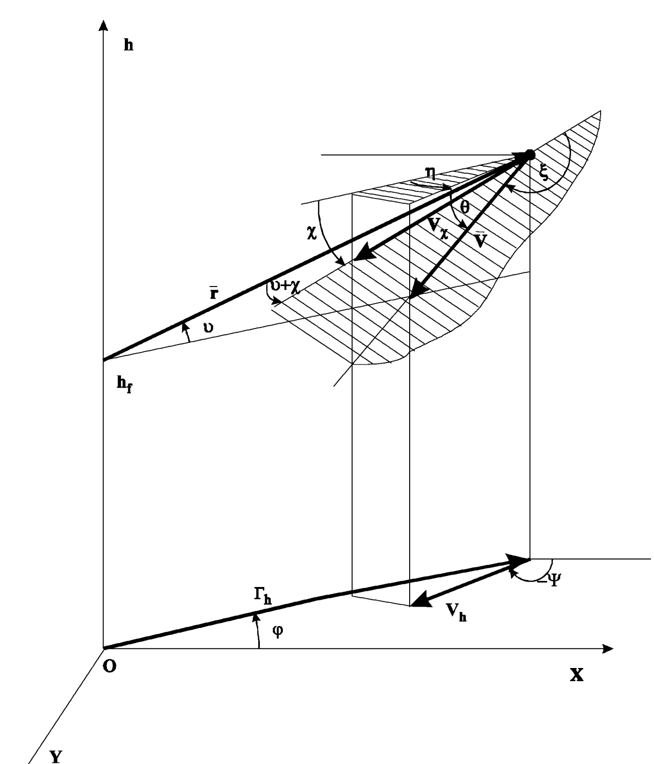

Fig. 1 Geometry of three-dimensional motion.

Three-Dimensional Guidance

Let us consider motion in the vertical plane. The kinematic equations for the vertical plane (Fig. 1) are

$$\dot{r} = V_\chi \cos(\chi + \vartheta) \tag{6a}$$

$$\dot{\vartheta} = (V_\chi / r)\sin(\chi + \vartheta) \tag{6b}$$

where

$$V_\chi = V\cos\xi \tag{7}$$

The proportional navigation law can be written as

$$\dot{\chi} = \lambda_V \dot{\vartheta} \tag{8}$$

where λ_V is a navigation ratio for the vertical plane.

Integrating this equation for ideal guidance (i.e., for $\lambda_V = \text{const}$) yields

$$\chi_f - \chi_0 = \lambda_V(\vartheta_f - \vartheta_0) \tag{9}$$

After substituting final values of $\chi_f = \theta_f$ and $\vartheta_f = -\theta_f$, we have

$$\lambda_V = -(\theta_f - \chi_0)/(\theta_f + \vartheta_0) \tag{10}$$

For the angles ξ, $\eta = \chi + \nu$, χ, and θ, this can be written

$$\sin\xi = \cos\theta\sin\eta \tag{11}$$

$$\sin\theta = \sin\chi\cos\xi \tag{12}$$

Suppose that the motions in vertical and horizontal planes are independent of one another. Thus, we can considered the angle of ξ as a constant. Differentiating Eq. (12) yields

$$\dot{\theta} = \cos\xi(\cos\chi/\cos\theta)\dot{\chi} \tag{13}$$

It is clear that for $\eta = \pm\pi$, 0, there is an equality of $\theta = \chi$.

For $|\eta| = \pi/2$, Eqs. (11) and (12) are indefinite. Therefore, the guidance for vertical plane should be start with $|\eta| \geq \pi/2 + \Delta\eta$, where $\Delta\eta$ is a margin for the elimination of the indefiniteness. The horizontal navigation rate λ_v is negative because the angles of χ and ϑ have the opposite rotation signs.

Also, as in case of the horizontal guidance, the navigation ratio λ_v must be computed periodically. It is evident that the acceptable state for vertical guidance must be satisfied to the following conditions: 1) $\chi + \vartheta \leq 0$ for $\vartheta \geq \theta_f$ or $\chi + \vartheta \geq 0$ for $\vartheta < \theta_f$ and 2) $|\lambda_v| > 2$. If these conditions are violated, a constrained motion (as examples, a climb or ascent or flight with $\dot{\theta} = 0$) is needed to satisfy the conditions.

Terminal Guidance of Lifting-Body Vehicle

The well-known equations of motion for an unpowered vehicle flying over a flat-Earth model are

$$\dot{h} = V\sin\theta \tag{14}$$

$$\dot{x} = V\cos\theta\cos\psi \tag{15}$$

$$\dot{y} = V\cos\theta\sin\psi \tag{16}$$

$$\dot{V} = -(D/m) - g\sin\theta \tag{17}$$

$$\dot{\theta} = (L/mV)\cos\sigma - (g/V)\cos\theta \tag{18}$$

$$\dot{\psi} = (L/mV\cos\theta)\sin\sigma \tag{19}$$

where

$$D = \tfrac{1}{2}\rho(h)SV^2C_D(M,\alpha), \qquad L = \tfrac{1}{2}\rho(h)SV^2C_L(M,\alpha)$$

where $\rho(h)$ is the air density as a function of altitude and $C_L(M,\alpha)$ and $C_D(M,\alpha)$ are the lift and drag coefficients as functions of the Mach number M and angle of attack α. We assume that lift and drag coefficients are trimmed values.

The guidance problem is as follows: given the model, Eqs. (14–19), of the vehicle's longitudinal and lateral dynamics, determine the controls, namely, the angles of attack α and of bank σ, as functions of the state, that steer the vehicle on a feasible trajectory passing through final point with desired velocity vector direction. To calculate the controls based on the required $\dot{\psi}$ and $\dot{\theta}$, the bank angle σ can be expressed as[7]

$$\tan\sigma = \frac{\dot{\psi}\cos\theta}{\dot{\theta} + (g/V)\cos\theta} \tag{20}$$

and the required lift coefficient can be determined as

$$C_L = (2m/\rho VS)\sqrt{(\dot{\psi}\cos\theta)^2 + [\dot{\theta} + (g/V)\cos\theta]^2} \tag{21}$$

The proposed method is applied to simulation of a lifting-body flight vehicle. As an example, we considered the HL-20 vehicle.[7] The HL-20 has a mass of 10,400 kg and reference area 26 m^2. Aerodynamics of the vehicle and atmospheric model are described in Ref. 7. Equations (14–19) are numerically integrated up to a final altitude h_f. The navigation ratios of λ_h and λ_V and the guidance constraints are computed at each integration point. The following assumptions are used for simulation of the trajectories:

1) If the current state does not belong to an accessible state for the horizontal guidance, then the maximum rate turn is needed (with a constraint of $\dot{\theta} \approx 0$) to an appropriate heading angle with $\lambda_h \geq 2.5$.

2) The beginning of vertical guidance corresponds to the conditions: $|\psi - \psi_f| \leq 15$ deg, $|\psi| \leq 15$ deg, and $|\lambda_V| \geq 2.2$.

3) At the final phase (for $r < 0.5$ km), the required angular derivatives are $\dot{\theta} \approx 0$ and $\dot{\psi} \approx 0$.

To illustrate the guidance law, two trajectories are computed. The initial states are given in Table 1.

The desired conditions are $h_f = 1$ km; $x_f, y_f = 0$; $\theta_f = -19$ deg; and $\psi_f = 180$ deg.

Figures 2 and 3 show the trajectories in the horizontal and vertical planes. The navigation ratios λ_V, λ_g, angles σ, and θ histories are presented in Figs. 4–7, respectively.

For the horizontal motion, the first trajectory included two phases. They are the forced turn and essentially guidance. The navigation

Table 1 Initial state parameters

Trajectory number	h_0, km	x_0, km	y_0, km	V_0, m/s	θ_0, deg	ψ_0, deg
1	10	35	−13	300	−11	175
2	10	14	−3	300	−23	0

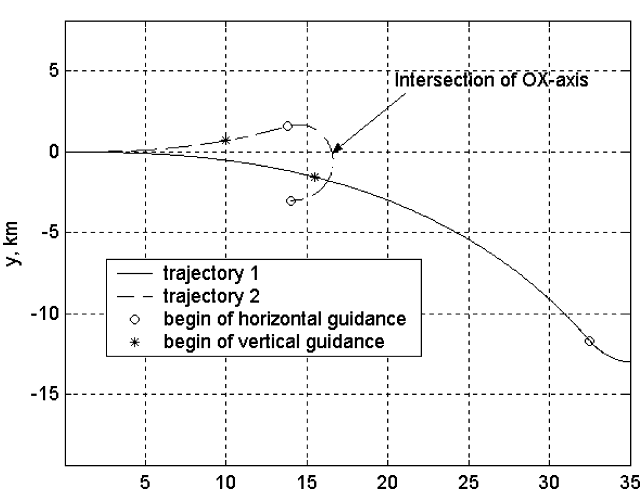

Fig. 2 Trajectories in horizontal plane.

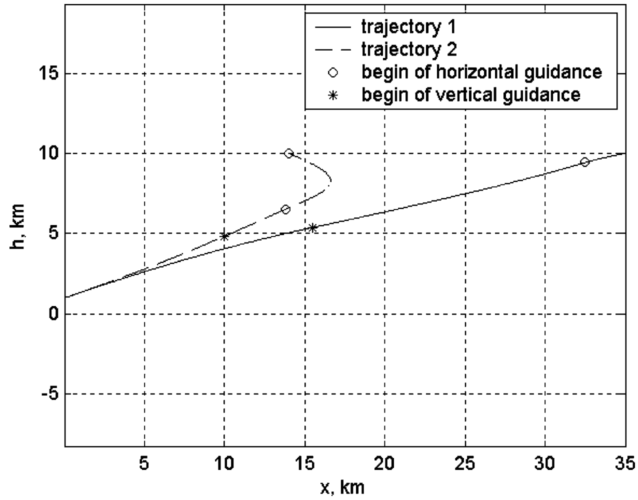

Fig. 3 Trajectories in vertical plane.

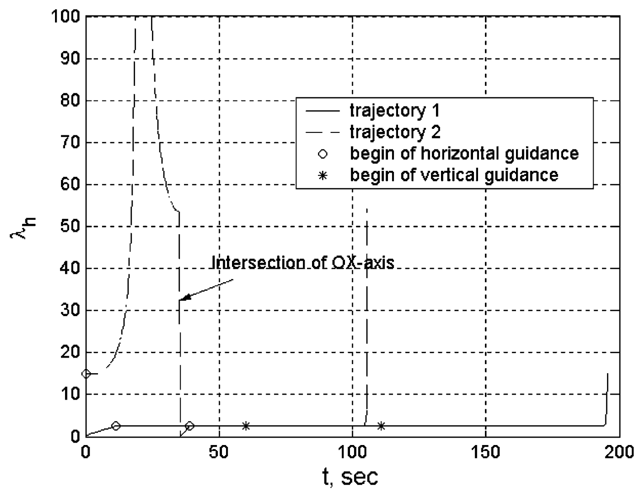

Fig. 4 Horizontal navigation ratio histories.

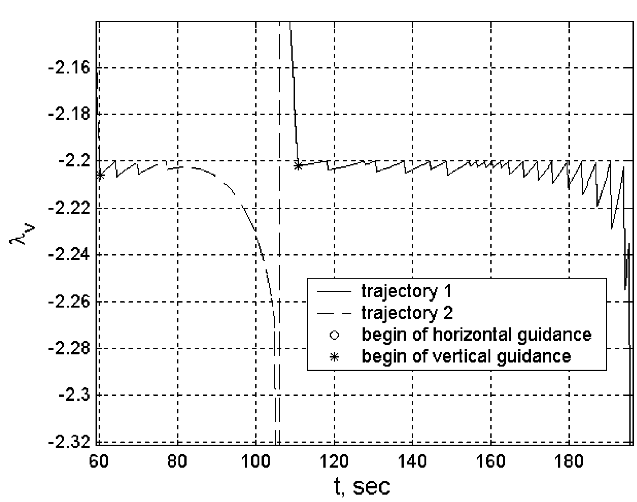

Fig. 5 Vertical navigation ratio histories at guidance phase.

ratios are stable at the corresponding guidance phases (Figs. 4 and 5). For the second trajectory, the horizontal guidance formally started at the initial point. However, the trajectory with the maximum allowable bank angle has no time to turn in the desired heading. Therefore, the trajectory intersected the OX axis, and the vehicle reached a state that is not allowable for the horizontal guidance law. The motion with maximum bank angle (for $\dot{\theta} \approx 0$) continued up to an acceptable state (Fig. 6). In this example, the horizontal guidance ratio is varied in a wide range (Fig. 4). For both examples the flight-path angles at

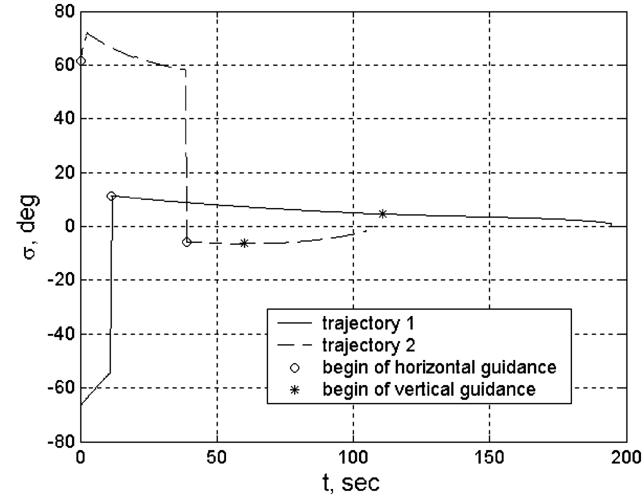

Fig. 6 Bank-angle histories.

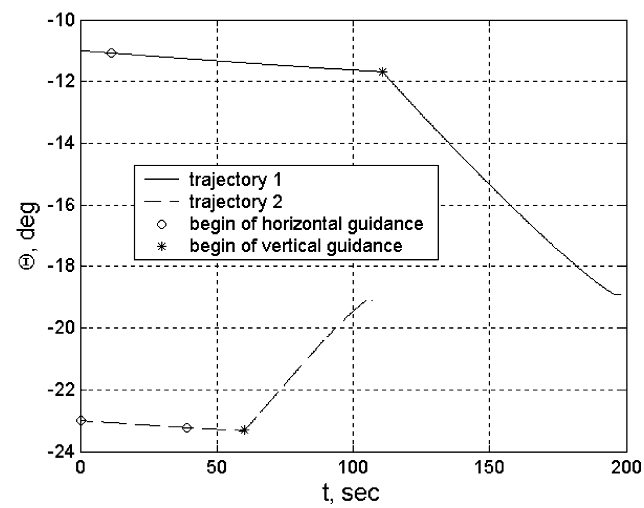

Fig. 7 Flight-path-angle histories.

the vertical guidance phases are changed approximately linearly to the time (Fig. 7).

Conclusions

In this study, we presented a novel guidance law for generation of vehicle trajectory with a specified heading at the desired final position. The key idea of the law is a modification of the well-known proportional navigation for the case of nonmoving target with the navigation ratio as a variable parameter depending on the current and desired final state. Perhaps the main advantage of the new guidance law is that it does not require a solution of a boundary problem for trajectories. In a sense, it is an explicit guidance method. The considered examples show that the method, in principle, can be used for the three-dimensional guidance. We believe that the proposed law is well suited as a part of an adaptive control to improve guidance quality of vehicles.

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